High-order discontinuous Galerkin scheme for coagulation/fragmentation equation

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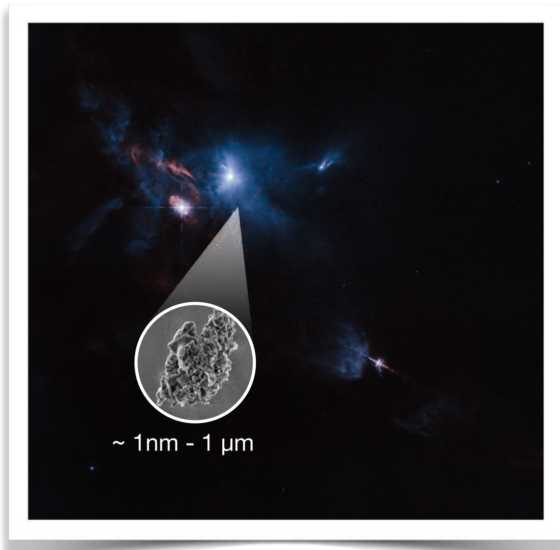
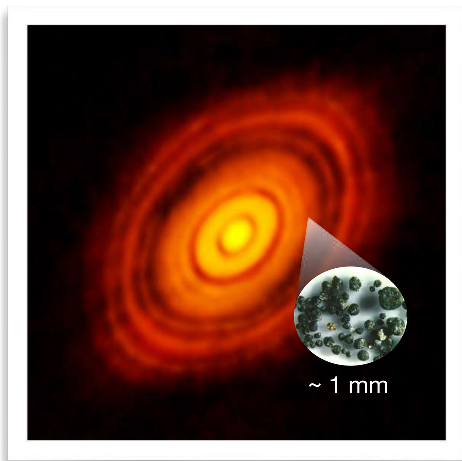
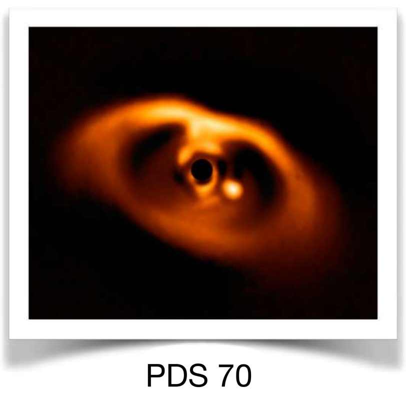
How can microscopic dust grains grow to form planet in less than 1 Myr ?

Taurus molecular cloud



HL Tau



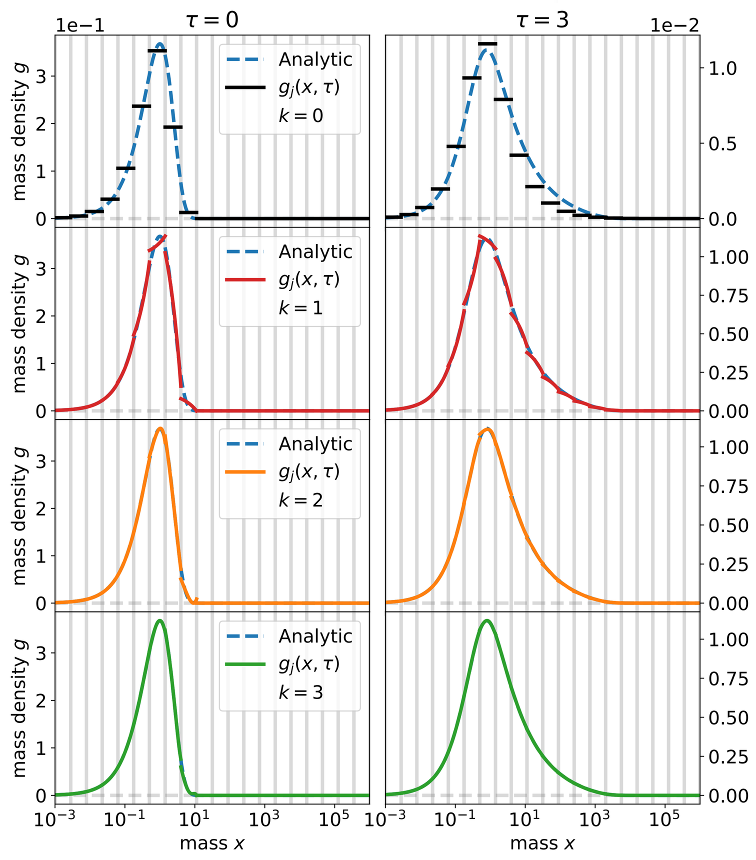


PDS 70

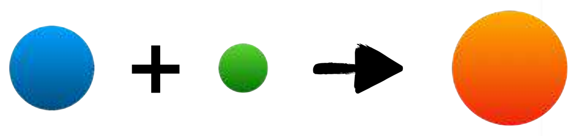
Goal

Coagulation

$$F[g](x,\tau) \equiv -\int_0^x \int_{x-u}^{\infty} \frac{K(u,v)}{v} g(u,\tau) g(v,\tau)\, dv\, du$$



- Test additive kernel $K(x,y) = x + y$
- ~0.1-1% in accuracy with 20 bins for order 3
- 9 orders of magnitude in mass



Lombart, M., Laibe, G., MNRAS, 501, 2021

Coagulation and fragmentation for 3D hydrodynamics codes:

- How to reach ~0.1-1% in accuracy with at maximum 20 dust sizes by spanning 12 orders of magnitudes in mass (grains from µm to mm)?
- How to design the solver to be computationally fast ?

PHANTOM

RAMSES

FARGO3D

Numerical obstacles

THE DISCONTINUOUS GALERKIN METHOD

$$\frac{\partial g(x,\tau)}{\partial \tau} + \frac{\partial F[g](x,\tau)}{\partial x} = 0$$

WITH

COALA

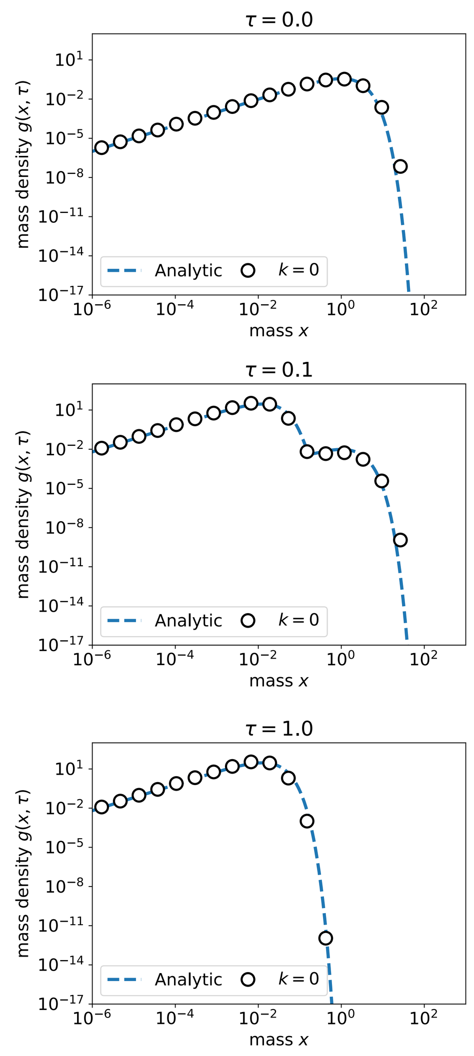
General non-linear fragmentation

Work in progress

NEW

$$F[g](x,\tau) \equiv \int_0^x \int_{x-u}^{\infty} \frac{K(u,v)}{v} g(u,\tau) g(v,\tau)\, dv\, du$$
$$-\frac{1}{2}\int_0^{x}\int_0^{\infty}\int_0^{\infty} \mathbb{1}_{u+v\geq x} \frac{w}{uv} b(w,u,v) K(u,v)\, g(u,\tau) g(v,\tau)\, dv\, du\, dw$$

- Test constant kernel $K(y,z) = 1$ and distribution of fragments $b(x,y,z) = \gamma^2 (y+z) e^{-\gamma x}$
  - Test with 20 bins order 0
  - 9 orders of magnitude in mass



Lombart, M., Hutchison, M., Lee, Y.-N., MNRAS, 517, 2022

Coagulation physical test

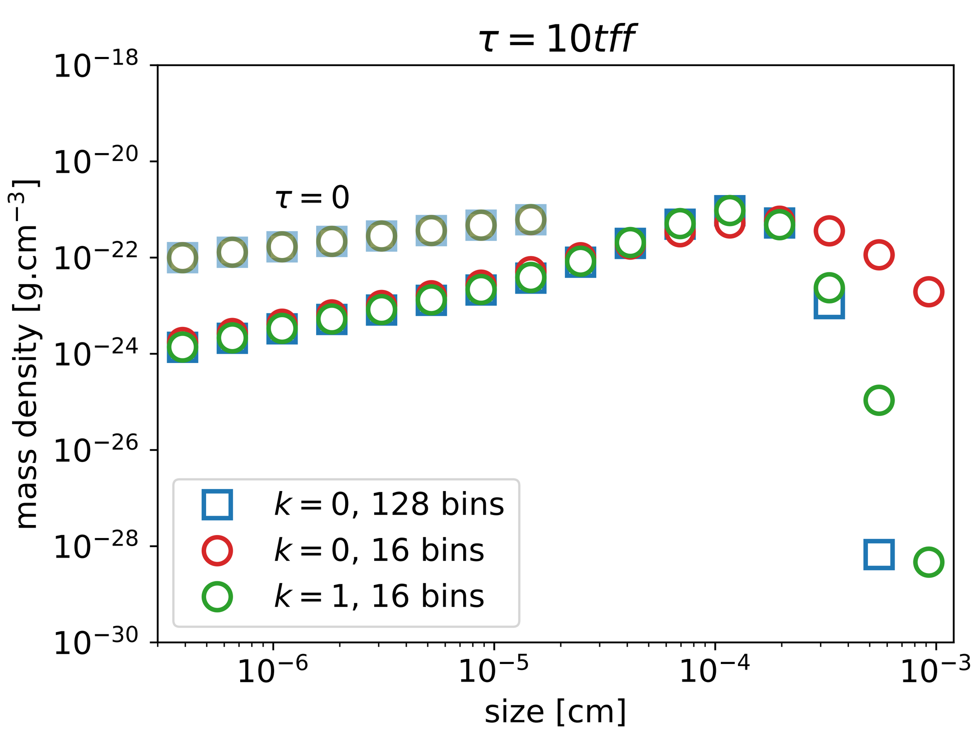
Work in progress

Hirashita & Li, 2013; Paruta et al., 2016

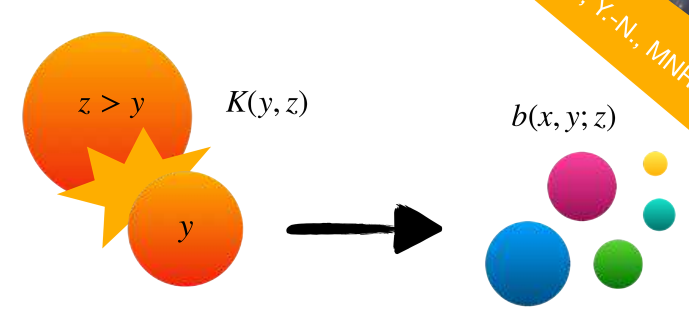
$$K(x,y) = \pi (x^{2/3} + 2x^{1/3}y^{1/3} + y^{2/3}) \Delta v(x,y)$$

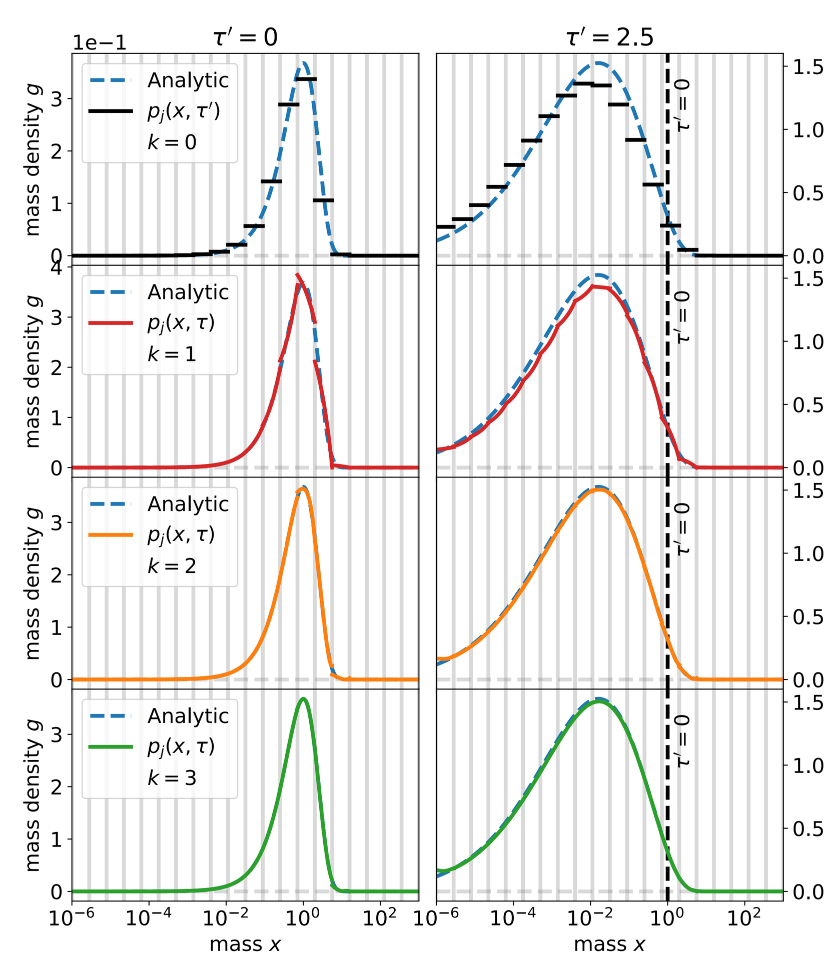
Coagulation in dense molecular cloud:

- Turbulence-driven differential velocities: $\Delta v \propto m_1^{1/6} + m_2^{1/6}$
- Initial condition: MRN distribution
- Reference solution: order 0 with 128 bins



Non-linear fragmentation





$$F[g](x,\tau) \equiv -\int_x^{\infty}\int_x^{\infty}\int_0^{x} \frac{w}{uv} b(w,u;v) K(u,v)\, g(u,\tau) g(v,\tau)\, dw\, du\, dv$$

- Test constant kernel $K(x,y) = 1$ and binary fragmentation $b(x,y) = 2/y$
- ~0.1-1% in accuracy with 20 bins for order 3
- 9 orders of magnitude in mass

The DG scheme solves efficiently and accurately the coagulation and fragmentation equations to be implemented in 3D gas/dust hydrodynamics codes.

Lombart, M., Hutchison, M., Lee, Y.-N., MNRAS, 517, 2022
Laibe, G., Lombart, M., MNRAS, 510, 2022
Lombart, M., Laibe, G., MNRAS, 501, 2021
Liu, H. Et al., 2019, SIAM J. Sci. Comput., 41(3)
Banasiak, J. Et al., CRC Press, 2019

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