The Athena++ Adaptive Mesh Refinement Framework: Multigrid Solvers for Self-Gravity

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Background

Self-gravity is one of the key physical processes in star and planet formation. As the Poisson equation is elliptic and the information propagates instantaneously, it is not trivial to achieve good performance and scalability on modern massively parallel supercomputers.

We have implemented a new self-gravity solver based on the full multigrid method on Athena++. The code is publicly available on Github (https://www.athena-astro.app/).

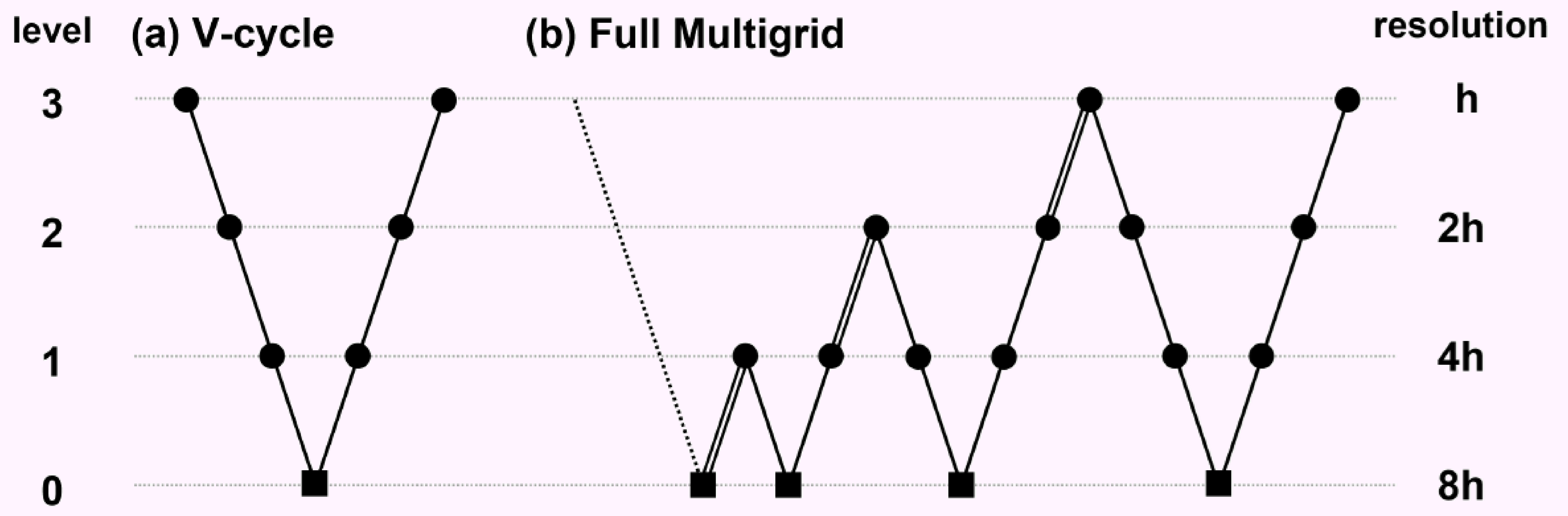
Algorithm and Implementation

The Poisson equation of self-gravity:

$$\nabla^2 \varphi = 4\pi G \rho$$

Its second-order discretization with spacing h:

$$\varphi_{i+1,j,k} + \varphi_{i-1,j,k} + \varphi_{i,j+1,k} + \varphi_{i,j-1,k} + \varphi_{i,j,k+1} + \varphi_{i,j,k-1} - 6\varphi_{i,j,k} = 4\pi G h^2 \rho_{i,j,k}$$



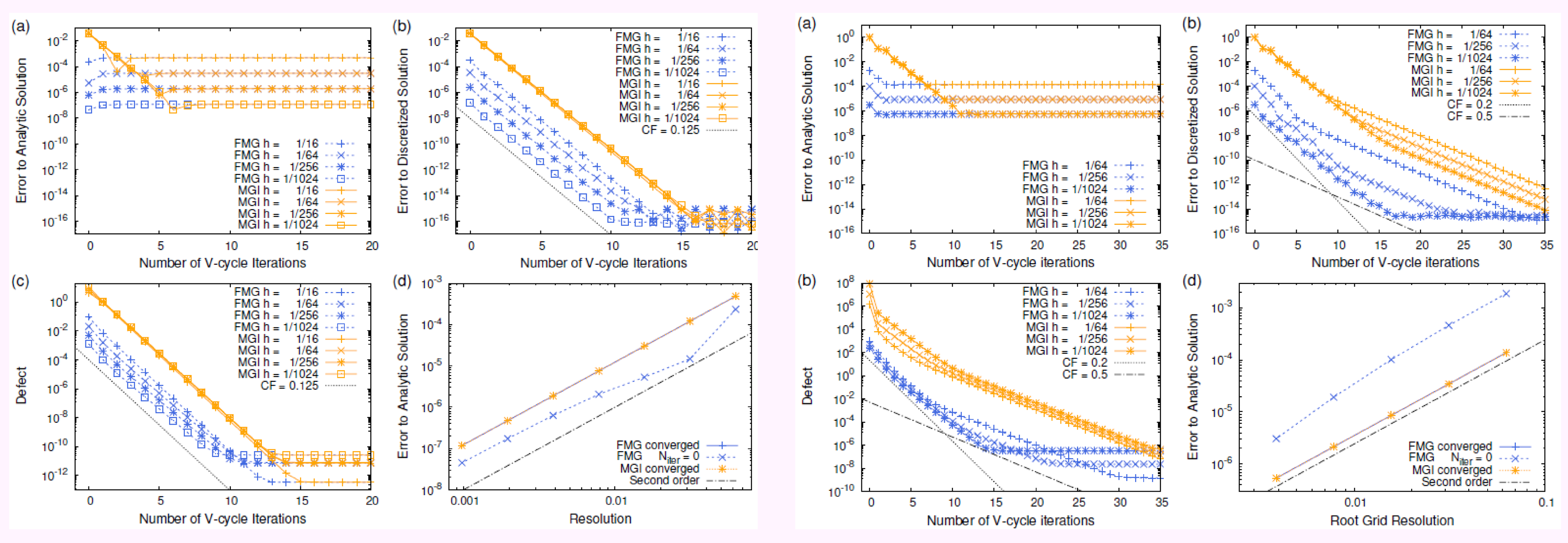
In the traditional V-cycle Multigrid (**MGI**) (a), a smoothing operator such as the Red-Black Gauss-Seidel smoother is applied on all the levels with different resolutions to damp errors of all wavelengths coherently.

In Full Multigrid (**FMG**) (b), we use the coarsest level solution as the initial guess for finer levels and improve the solution using the V-cycle Multigrid repeatedly. Once it reaches the finest level, we apply the V-cycles until a convergence threshold is satisfied. This method is superior as it does not require any initial guess.

Our implementation supports AMR. At level boundaries, we use the mass-conservation formula (Feng et al. 2018) to ensure consistency between the levels.

The code is efficiently parallelized using Athena++'s TaskList dynamic execution model.

Accuracy and Performance



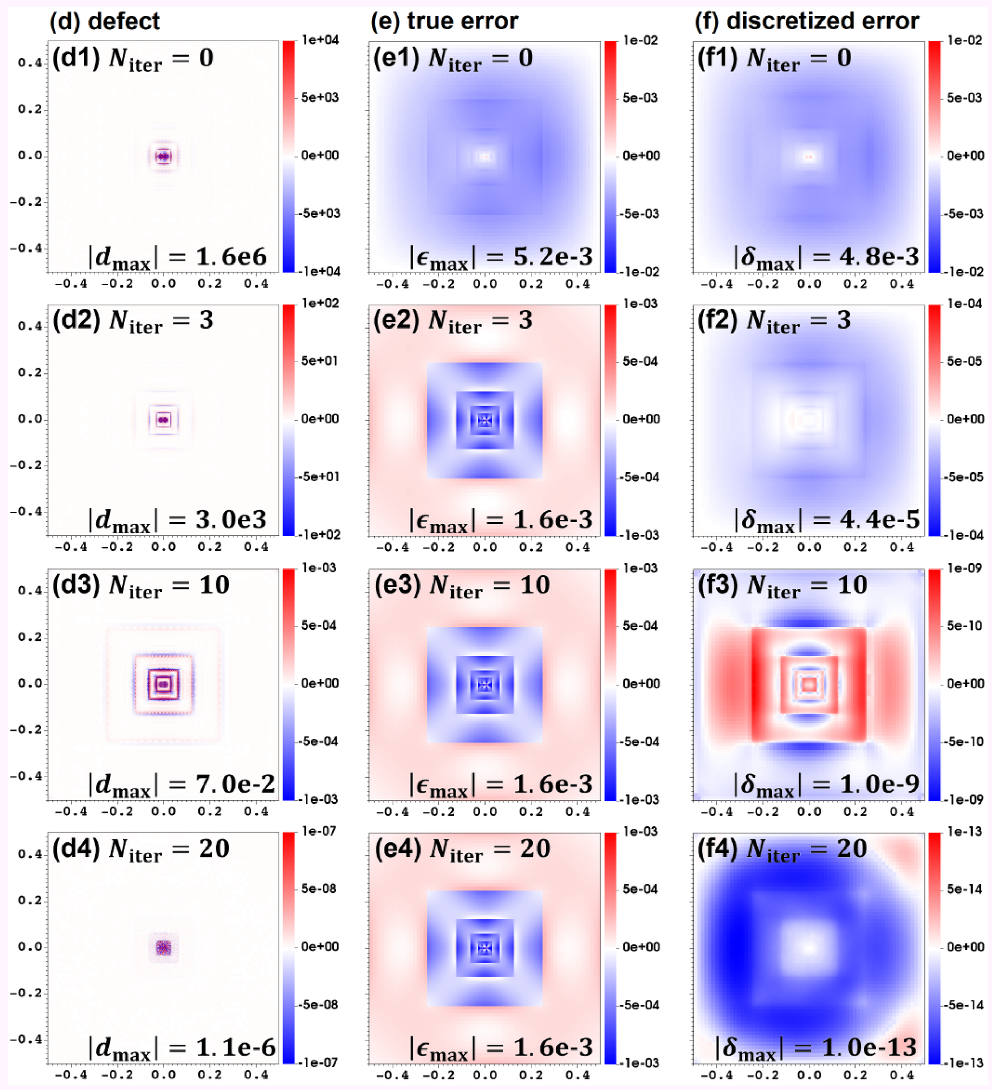
↑ Left: convergence of sin waves on uniform grid
Right: convergence of binary potential on AMR

(a) error to the analytic solution, (b) error to the fully converged solution, (c) defect, (d) convergence.

Blue: **FMG**, **Orange**: **MGI** with naïve initial guess

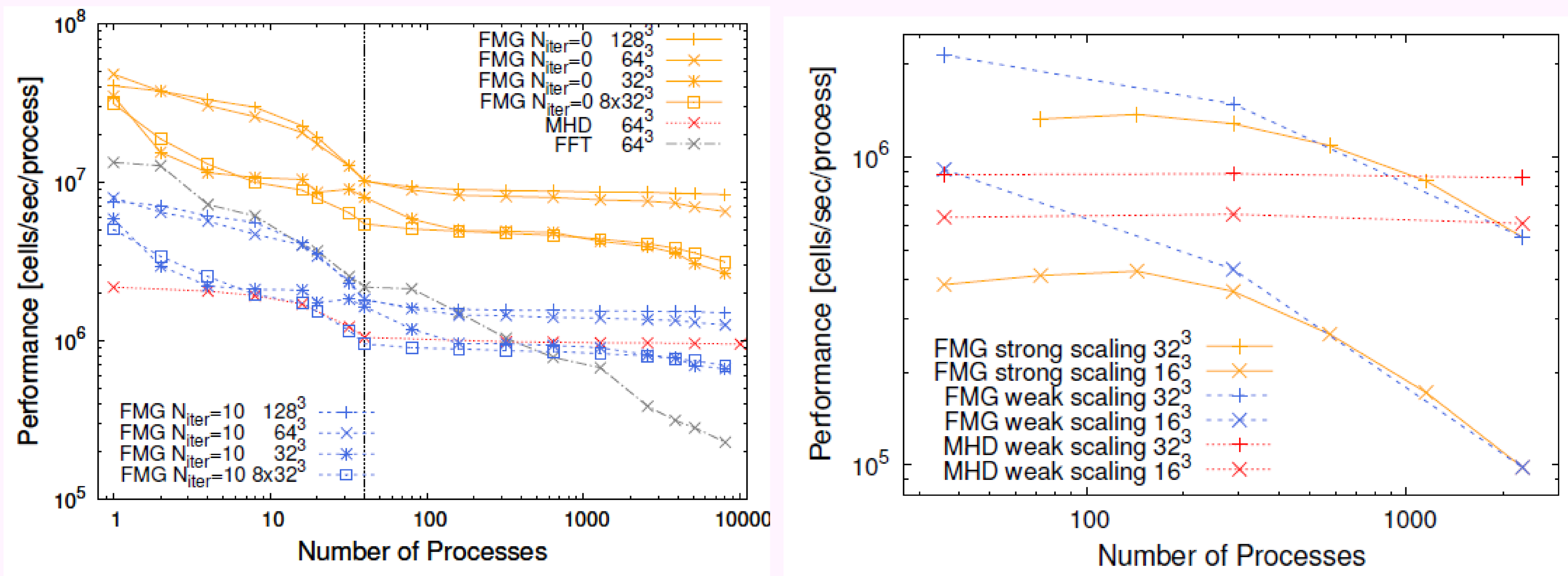
- Full Multigrid always outperforms traditional Multigrid
- Error to the analytic solution quickly saturates
- Second-order accuracy is achieved in all the cases

Accuracy and Performance (cont.)



← Convergence behavior of the AMR binary test.
Left: Distribution of defect
Center: error to the analytic solution
Right: error to the fully-converged solution

- Error to the analytic solution saturates quickly
- Persisting noises near level boundaries cause slow convergence



↑ Left: Weak-scaling performance on uniform grid
Right: Performance of the AMR binary test.

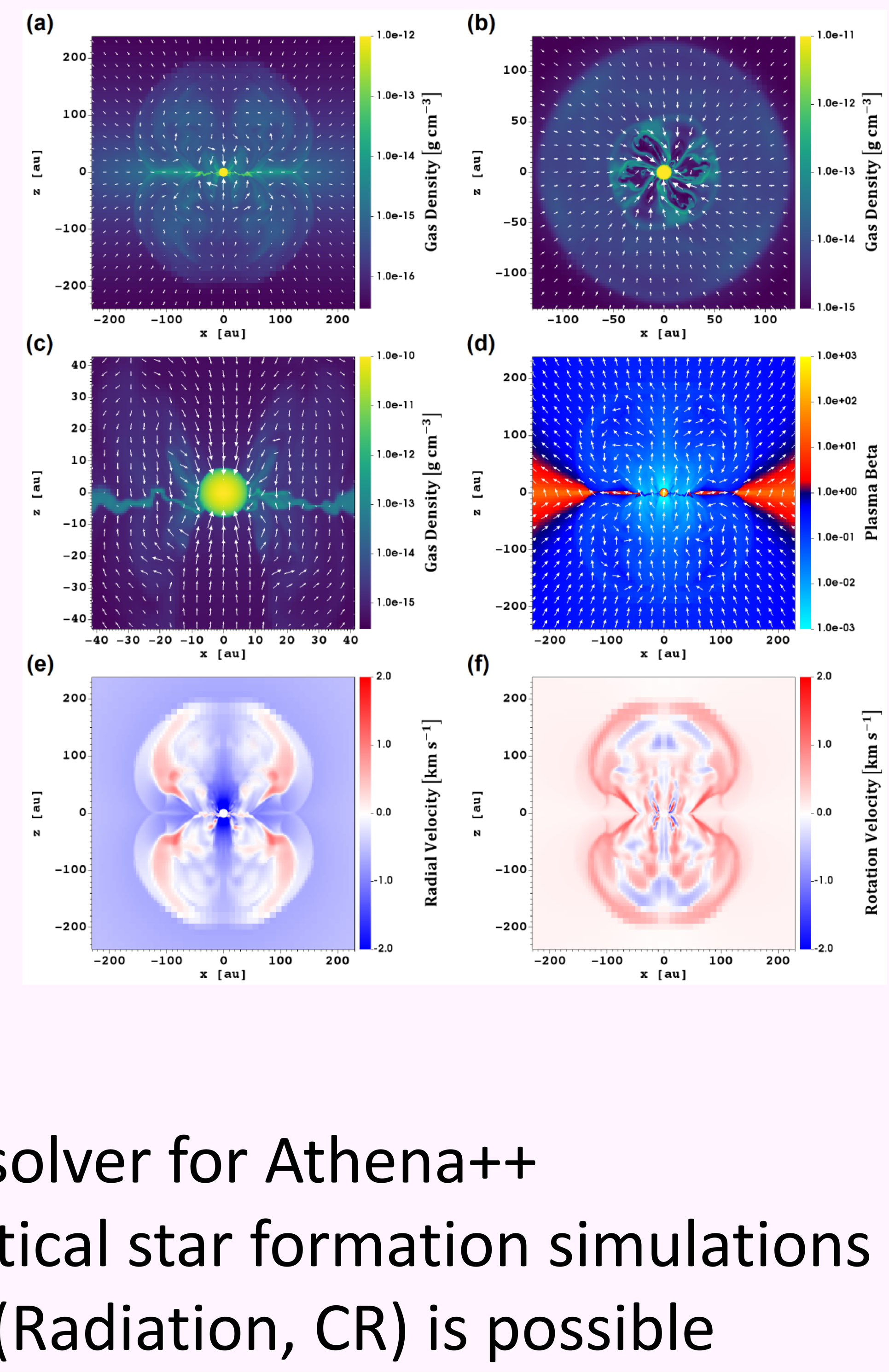
Blue: **FMG** + 10 V-cycles, **Orange**: **FMG** alone,
Red: **MHD**, **Gray**: **FFT**

- FMG scales well and as fast as MHD on uniform grids.
- Multigrid scales better than FFT because computational complexity of Multigrid is $O(N)$, while FFT is $O(NlogN)$
- Scaling on AMR is not excellent, but still is reasonable compared to other public codes (e.g. FLASH).

Demonstration and Summary

AMR simulation of collapse of a rotating magnetized molecular cloud core with AMR and barotropic EOS →

- First core formation
- Angular mom. transport by magnetic braking
- Bipolar outflows
- Consistent results with previous simulations
- ~ 5,000 core hours, or <1.3 days using 160 cores



Summary:

- New Multigrid self-gravity solver for Athena++
- It can be applicable to practical star formation simulations
- Extension to other physics (Radiation, CR) is possible

As I am one of the LOC co-chairs of this conference, I may be too busy to enjoy discussion with you. Please give me your questions and comments on Mattermost or by email. I hope you enjoy the conference as well as your stay in Kyoto.

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